

Creating macroscopic atomic EPR states from Bose condensates

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We present a scheme for creating quantum entangled atomic states through the coherent spin-exchange collision of a spinor Bose-Einstein condensate. The state generated possesses macroscopic Einstein-Podolsky-Rosen correlation and the fluctuation in one of its quasi-spin components vanishes. We show that an elongated condensate with large aspect ratio is most suitable for creating such a state.

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Quantum entanglement lies at the heart of the profound difference between quantum mechanics and classical physics [1]. The entanglement between the states of space-like separated particles is the fundamental reason for the violation of Bell inequality, and causes many of the “paradoxes” of quantum physics. In recent years, there has been an interesting maturing of the discussions of entanglement away from the foundations of quantum mechanics and to “applications” in the emerging field of quantum information processing.

A majority of the experimental realizations of quantum entanglement to date involve the creation of entangled photon pairs. Although ideal as carriers of quantum information, photons are however normally difficult to store for extended periods of time, in contrast to massive particles. To overcome this difficulty, progresses have been made to generate correlated atom-photon pairs [2,3]. Recently, much attention has also been paid to quantum correlated atomic systems, particularly non-classical multi-atom states [4–7], as these systems have important applications in quantum measurement beyond the “standard quantum limits” as well as in quantum computation.

There have already been several proposals to create entangled atomic ensembles [8,9] and one of them has recently been demonstrated experimentally [10]. All of these schemes rely on mapping the nonclassical properties of electromagnetic waves, e.g., squeezed light, onto the state of an atomic system. In this Letter, we show that by taking advantage of coherent spin-exchange ultracold collisions, one can generate macroscopic atomic Einstein-Podolsky-Rosen (EPR) states [11] from a spinor Bose-Einstein condensate without the need of nonclassical light fields.

We proceed by first giving the general idea of the proposed technique, and then turn to a more detailed theoretical discussion. Our scheme is illustrated in Fig. 1. A spinor Bose-Einstein condensate consisting of a dilute

$F = 1$ atomic sample is initially polarized such that only the spin-0 hyperfine ground state is populated at time $t = 0$. Binary spin-exchange interaction then convert the spin-0 atoms into pairs of spin- (± 1) atoms. The irreversibility of such a process is provided by shifting the energy of the spin-0 state above that of the spin- (± 1) states (which can be achieved using the ac Stark shift provided by far off-resonant laser light [12]). As a result of this detuning, the phase-matching condition, i.e., conservation of momentum and energy, ensures that the resultant atoms in the pair move in opposite directions away from each other and escape the trap. Quantum entanglement results from our ignorance about which of the two escaping atoms is in the spin- $(+1)$ state and which has spin- (-1) .

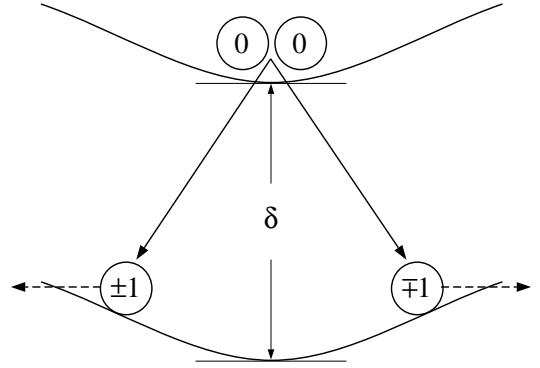


FIG. 1. Entanglement scheme: A spin-0 condensate is initially prepared. Spin-exchange interaction creates spin- (± 1) atom pairs whose energy level is shifted below that of spin-0 atom by an amount of $\hbar\delta$. This excess energy is transferred into the kinetic energies of spin- (± 1) atoms which escape the trap.

We now turn to a detailed analysis of this system. At $t = 0$, a condensate of N_0 spin-0 atoms is confined in an optical dipole trap. An additional off-resonant optical field is used to shift the energy of the spin-0 state above those of the spin- (± 1) states by an amount $\hbar\delta$ (see Fig. 1). The spatial wave function of the condensate, $\varphi(\mathbf{r})$ is determined by the stationary Gross-Pitaevskii equation.

At $t > 0$, the spin- (± 1) states start being populated by the spin-exchange interaction

$$H = \lambda_a \int d\mathbf{r} \hat{\psi}_{+1}^\dagger(\mathbf{r}, t) \hat{\psi}_{-1}^\dagger(\mathbf{r}, t) \hat{\psi}_0(\mathbf{r}, t) \hat{\psi}_0(\mathbf{r}, t) + h.c. \quad (1)$$

where λ_a is a constant related to the s -wave scattering lengths associated with the hyperfine levels involved, and

$\hat{\psi}_\alpha$ is the boson annihilation operator for spin- α atoms. The effect of atomic recoil during this process is to transfer the excess energy $\hbar\delta$ into the kinetic energy of the spin- (± 1) atoms. Therefore, for the short time scale where the propagation of (± 1) atoms can be neglected, we may expand the boson field operators as

$$\hat{\psi}_0(\mathbf{r}, t) = \varphi(\mathbf{r}) e^{-i\delta t} \hat{c}_0(t) \quad (2)$$

$$\hat{\psi}_{\pm 1, \mathbf{q}}(\mathbf{r}, t) = \varphi(\mathbf{r}) e^{i(\mathbf{q} \cdot \mathbf{r} - \omega_q t)} \hat{c}_{\pm 1, \mathbf{q}}(t), \quad (3)$$

where $\omega_q \equiv \hbar|\mathbf{q}|^2/(2m)$, and the operators $\{\hat{c}_\mu\}$ obey the boson commutation relations $[\hat{c}_\mu, \hat{c}_\nu^\dagger] = \delta_{\mu, \nu}$. With these expansions, Hamiltonian (1) may be reexpressed as

$$H = \kappa \int \int d\mathbf{q} d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') e^{i\Delta_{q, q'} t} \hat{c}_{+1, \mathbf{q}}^\dagger \hat{c}_{-1, \mathbf{q}'}^\dagger \hat{c}_0 \hat{c}_0 + h.c., \quad (4)$$

where $\Delta_{q, q'} \equiv (\omega_q + \omega_{q'} - 2\delta)$, $\kappa \equiv \lambda_a V^2/(2\pi)^6$ with V being the quantization volume, and

$$\rho(\mathbf{q}, \mathbf{q}') = \int d\mathbf{r} |\varphi(\mathbf{r})|^4 e^{-i(\mathbf{q} + \mathbf{q}') \cdot \mathbf{r}}. \quad (5)$$

Eq. (4) is reminiscent of the Hamiltonian describing parametric down conversion processes in nonlinear and quantum optics. As is well known, these processes lead to squeezing and to the generation of entangled photon pairs.

For short enough interaction times, the population of the sidemodes (± 1) remain small compared to N_0 . In this regime, we neglect the depletion of the spin-0 state and treat \hat{c}_0 as a c -number c_0 such that $|c_0|^2 = N_0$. We can furthermore neglect those terms in the Hamiltonian (4) that describe atom-atom interactions involving only the spin- (± 1) states. Under these assumptions, the Heisenberg dynamics of the operators $\hat{c}_{\pm 1, \mathbf{q}}$ resulting from the interaction Hamiltonian (4) simplifies to

$$\frac{d}{dt} \hat{c}_{+1, \mathbf{q}} = -i\kappa \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') e^{i\Delta_{q, q'} t} \hat{c}_{-1, \mathbf{q}'}^\dagger c_0^2, \quad (6)$$

$$\frac{d}{dt} \hat{c}_{-1, \mathbf{q}} = -i\kappa \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') e^{i\Delta_{q, q'} t} \hat{c}_{+1, \mathbf{q}'}^\dagger c_0^2. \quad (7)$$

To solve these equations, we first formally integrate Eq. (7) to get

$$\begin{aligned} \hat{c}_{-1, \mathbf{q}}(t) &= \hat{c}_{-1, \mathbf{q}}(0) - i\kappa c_0^2 \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') \int_0^t d\tau e^{i\Delta_{q, q'} \tau} \\ &\quad \times \hat{c}_{+1, \mathbf{q}'}^\dagger(t - \tau) \\ &\approx \hat{c}_{-1, \mathbf{q}}(0) - i\kappa c_0^2 \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') \delta(\Delta_{q, q'}) \hat{c}_{+1, \mathbf{q}'}^\dagger(t) \end{aligned} \quad (8)$$

where the Markov approximation has been invoked. Inserting Eq. (8) into Eq. (6), we obtain

$$\frac{d}{dt} \hat{c}_{+1, \mathbf{q}} = \frac{N_0^2}{2} G_{\mathbf{q}} \hat{c}_{+1, \mathbf{q}} + \hat{f}_{\mathbf{q}}^\dagger(t) \quad (9)$$

where we have defined the gain parameter

$$G_{\mathbf{q}} = 2\pi\kappa^2 \int d\mathbf{q}' |\rho(\mathbf{q}, \mathbf{q}')|^2 \delta(\Delta_{q, q'}) \quad (10)$$

and the noise operator

$$\hat{f}_{\mathbf{q}}^\dagger(t) = -i\kappa c_0^2 \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') e^{i\Delta_{q, q'} t} \hat{c}_{-1, \mathbf{q}'}^\dagger(0),$$

whose correlation functions are given in the Markov approximation by

$$\begin{aligned} \langle \hat{f}_{\mathbf{q}}^\dagger(t) \hat{f}_{\mathbf{q}}(t') \rangle &= 0, \\ \langle \hat{f}_{\mathbf{q}}(t) \hat{f}_{\mathbf{q}'}^\dagger(t') \rangle &= N_0^2 G_{\mathbf{q}} \delta(\mathbf{q} - \mathbf{q}') \delta(t - t'). \end{aligned}$$

It is this noise operator that triggers the populating of spin- $(+1)$ state from quantum fluctuations. In deriving Eq. (9), we have used the approximation $\rho(\mathbf{q}, \mathbf{q}') \rho^*(\mathbf{q}', \mathbf{q}'') \approx |\rho(\mathbf{q}, \mathbf{q}')|^2 \delta(\mathbf{q} - \mathbf{q}'')$ and neglected the principal part associated with the definition of the δ -function.

Following a similar procedure, we can derive the equation of motion for $\hat{c}_{-1, \mathbf{q}}$ as

$$\frac{d}{dt} \hat{c}_{-1, \mathbf{q}} = \frac{N_0^2}{2} G_{\mathbf{q}} \hat{c}_{-1, \mathbf{q}} + \hat{g}_{\mathbf{q}}^\dagger(t), \quad (11)$$

where

$$\hat{g}_{\mathbf{q}}^\dagger(t) = -i\kappa \int d\mathbf{q}' \rho(\mathbf{q}, \mathbf{q}') e^{i\Delta_{q, q'} t} \hat{c}_{+1, \mathbf{q}'}^\dagger(0) c_0^2. \quad (12)$$

At this level of approximation, which neglects as we recall the depletion of the spin-0 mode, the Heisenberg equations of motion (9) and (11) are linear. They can readily be integrated to give

$$\hat{c}_{+1, \mathbf{q}}(t) = \mathcal{G}_{\mathbf{q}}(t) \hat{c}_{+1, \mathbf{q}}(0) + \int_0^t d\tau \mathcal{G}_{\mathbf{q}}(\tau) \hat{f}_{\mathbf{q}}^\dagger(t - \tau) \quad (13)$$

$$\hat{c}_{-1, \mathbf{q}}(t) = \mathcal{G}_{\mathbf{q}}(t) \hat{c}_{-1, \mathbf{q}}(0) + \int_0^t d\tau \mathcal{G}_{\mathbf{q}}(\tau) \hat{g}_{\mathbf{q}}^\dagger(t - \tau) \quad (14)$$

where $\mathcal{G}_{\mathbf{q}}(t) \equiv \exp(N_0^2 G_{\mathbf{q}} t/2)$. From these we can calculate the population in modes $\{\pm 1, \mathbf{q}\}$:

$$N_{\pm 1, \mathbf{q}} = \langle \hat{c}_{\pm 1, \mathbf{q}}^\dagger \hat{c}_{\pm 1, \mathbf{q}} \rangle = \exp(N_0^2 G_{\mathbf{q}} t) - 1$$

It is also straightforward to calculate the correlation function

$$\begin{aligned} \mathcal{C}_{\mathbf{q}, \mathbf{q}'} &\equiv \langle \hat{c}_{-1, \mathbf{q}} \hat{c}_{+1, \mathbf{q}'} \rangle \\ &= -i\kappa \mathcal{G}_{\mathbf{q}}(t) c_0^2 \rho_{\mathbf{q}}(\mathbf{q}') \frac{\mathcal{G}_{\mathbf{q}'}(t) - e^{i\Delta_{q, q'} t}}{N_0^2 G_{\mathbf{q}'} / 2 - i\Delta_{q, q'}} \end{aligned} \quad (15)$$

The fact that the ± 1 modes are correlated implies that the two spin states (± 1) are entangled. It is obviously desirable that a spin- (-1) atoms with momentum $\hbar\mathbf{q}$ be correlated to a spin- $(+1)$ atom with well defined momentum $\hbar\mathbf{q}'$.

From the definition (5) of $\rho(\mathbf{q}, \mathbf{q}')$, we conclude that as long as the spatial size of the condensate wave function is much larger than the reciprocal length $1/|\mathbf{q}|$ and $1/|\mathbf{q}'|$, $\rho(\mathbf{q}, \mathbf{q}')$ is approximately proportional to a delta-function, $\rho(\mathbf{q}, \mathbf{q}') \rightarrow \delta(\mathbf{q} + \mathbf{q}')$. In other words, under this condition the two correlated atoms resulting from a spin-changing collision move in opposite directions. Additionally, the particle momenta $|\mathbf{q}|$ and $|\mathbf{q}'|$ have to satisfy the conservation of energy condition $\omega_q + \omega_{q'} - 2\delta \approx 0$, a condition that can be met for a large light shift $\hbar\delta$. We observe that in addition, a large energy shift $\hbar\delta$ is also required to produce spin-(± 1) atoms with sufficiently large kinetic energy to escape the trap. As already mentioned, this is required to prevent them from undergoing a collision resulting in a pair of spin-0 atoms.

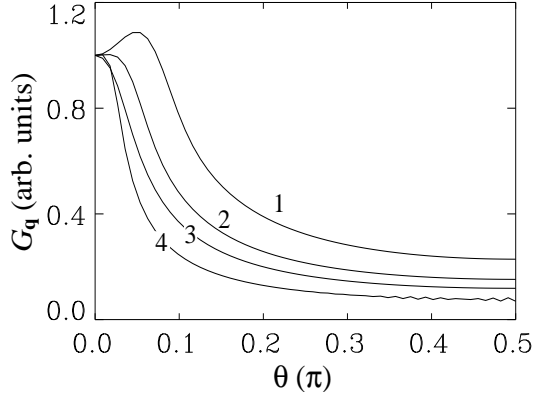


FIG. 2. The gain parameter $G_{\mathbf{q}}$ versus θ , for a Gaussian and cylindrically symmetric condensate wave function of the form $\varphi(\mathbf{r}) \propto \exp[-z^2/(2\sigma_z^2) - (x^2 + y^2)/(2\sigma_\perp^2)]$. θ is the angle between \mathbf{q} and the z -axis. In the calculation, we set $q = |\mathbf{q}| = \sqrt{2m\delta/\hbar}$. Curve 1: $\sigma_z = 10$, $q = 10$; curve 2: $\sigma_z = 10$, $q = 20$; curve 3: $\sigma_z = 10$, $q = 40$; curve 4: $\sigma_z = 20$, $q = 40$. The units for σ_z and q are σ_\perp and $1/\sigma_\perp$, respectively.

In general, it is not sufficient to just produce entangled atomic pairs. Rather, one would like to subsequently store them, e.g. in a dipole trap. It is desirable for this purpose to achieve a high degree of directionality in the generated atoms, so that they have a narrow enough angular distribution. To see how this can be achieved, let us take a closer look at the gain parameter $G_{\mathbf{q}}$ appearing in Eq. (10). The expression of $G_{\mathbf{q}}$ is reminiscent of a similar gain parameter encountered in the study of super-radiant scattering from a condensate [13,14]. It has been shown in that context that for a spatially anisotropic condensate, the largest gain occurs along the longest dimension of the condensate [14]. The same conclusion can be reached in the present case. Fig. 2 illustrates the gain along different directions for the case of a cylindrically symmetric condensate, for various light shifts $\hbar\delta$ and aspect ratios. For simplicity, we choose $q = |\mathbf{q}| = \sqrt{2m\delta/\hbar}$, and assume that the condensate has a Gaussian shape. Fig. 2 illustrates quite clearly that a smaller angular distribution of emitted atoms is obtained for larger as-

pect ratios and larger q . Thus for a strongly elongated cigar-shaped condensate, the matter-wave modes along the long axis, which have the largest gain coefficient $G_{\mathbf{q}}$, will typically deplete all the condensate atoms before the population of the off-axis modes can significantly build up. As a consequence of mode competition, the emission of the spin-(± 1) atoms is therefore largely confined to two narrow cones at the two ends of the cigar-shaped condensate.

From this discussion we conclude that in order to experimentally realize the proposed scheme, one should first create an elongated spin-0 condensate with a large light shift $\hbar\delta$. Spin-exchange interactions then generate pairs of spin-(± 1) atoms flying in opposite directions along its long axis. These atoms can be subsequently captured by two traps located at opposite sides of the original trap. Eventually, the spin-0 condensate is depleted, with two new ensembles of pair-wise entangled atoms stored inside the side traps. We emphasize that although each trap contains both spin-(+1) and spin-(−1) atoms, these cannot undergo subsequent spin-exchange collisions to produce spin-0 atoms, since this process does not satisfy momentum-energy conservation.

The spin-(± 1) atoms being created in pairs, we know for sure that taken together, the two ensembles must contain equal number of spin-(+1) and spin-(−1) atoms — although how many spin-(+1) and spin-(−1) atoms are in each ensemble is unknown. In the Schrödinger picture, such a state may be expressed as

$$|\Psi\rangle = \sum_{m=-N/2}^{N/2} a_m \left| \frac{N}{2}, m \right\rangle_l \left| \frac{N}{2}, -m \right\rangle_r, \quad (16)$$

where $N/2$ is the total number of atoms in each of the two “left” and “right” side traps, labeled by l and r , respectively. The integers m and $-m$ represent the difference in numbers of atoms in the spin states (+1) and (−1) in each of the two traps.

Introducing the z -component of the quasi-spin operator

$$\hat{L}_z^{(i)} = \hat{N}_{+1}^{(i)} - \hat{N}_{-1}^{(i)},$$

where $\hat{N}_{\pm 1}^{(i)}$ is the number operator for state-(± 1) in ensemble i and $i = l, r$, we have that

$$\hat{L}_z^{(i)} \left| \frac{N}{2}, m \right\rangle_i = m \left| \frac{N}{2}, m \right\rangle_i.$$

Since the explicit expressions of the coefficients a_m in (16) are unknown, so are the expectation value and variance for $\hat{L}_z^{(i)}$. However, a simple calculation shows that

$$\begin{aligned} \langle \hat{L}_z \rangle &= 0, \\ (\Delta \hat{L}_z)^2 &= 0, \end{aligned}$$

where $\hat{L}_z \equiv \hat{L}_z^{(l)} + \hat{L}_z^{(r)}$ is the z -component of the *total* quasi-spin operator. Hence, although the variance of the

$\hat{L}_z^{(i)}$ may be large for the individual ensembles, the variance for the whole system vanishes. In other words, taken as a whole the two ensembles represent a maximally spin-squeezed state. This should be contrasted to the case of N independent atoms in the state $(|+1\rangle + |-1\rangle)^N$, for which one finds $(\Delta\hat{L}_z)^2 = N/4$.

We note that if we randomly pick one atom each from the two side-traps for an atomic ensemble prepared in state (16), then their degree of entanglement is only of order $1/N$. This is because although the atoms are created in pairs, we cannot tell which particular pairs of atoms are entangled. It is only through the collective spin measurement that the quantum entanglement can be revealed. The observation of such an macroscopic entanglement can be carried out with the technique of spectroscopic detection of collective spin noise at the quantum level described in Refs. [10,15]. In practice, the state (16) has to be averaged over the statistical distribution of the total particle number N . However, as noted in Ref. [8], such fluctuations do not affect the entanglement significantly for large numbers of atoms.

In conclusion, we have proposed and analyzed a simple scheme to create a macroscopic EPR-correlated atomic state. Such a state possesses a nonlocal entanglement and is maximally squeezed, in the sense that the fluctuations of the z -component of its quasi-spin vanish. Hence we believe that this system will have important applications in precision measurement as well as in fundamental physics such as the test of nonlocality in macroscopic quantum systems. Our study shows that an elongated spinor condensate with large aspect ratio and large energy difference between spin-0 and spin- (± 1) states is the best candidate to create such a state. The correlations between the atomic ensembles arise from the nonlinear atom-atom interaction amongst the condensate atoms. This distinguishes our work from other proposals with a similar goal, where the correlations between atoms are transferred from EPR-correlated light fields. As a consequence, our scheme can deal with strictly ground state hyperfine atomic states. This is of considerable advantage, since the entanglement of the kind described here is therefore robust against decoherence and immune from the quantum fluctuations caused by the electromagnetic vacuum field modes, which limit the degree of entanglement and spin squeezing [8,10].

Note: Upon completion of our work, we noticed a preprint paper by Sørensen *et al.* [16] in which the possibility of creating squeezed spin state with Bose condensates is investigated. Our work differs from theirs as the atomic ensembles we studied are spatially separated with nonlocal EPR correlation, while theirs does not possess this property.

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